

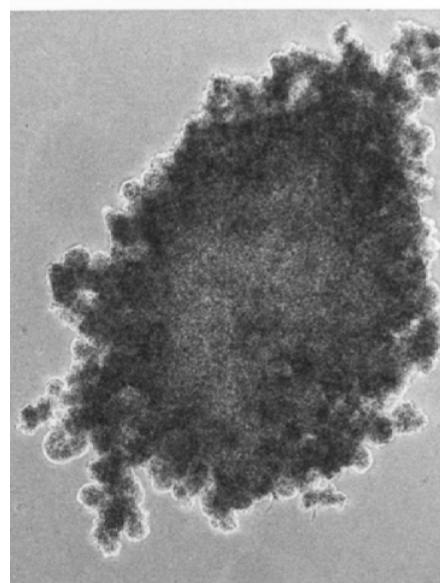
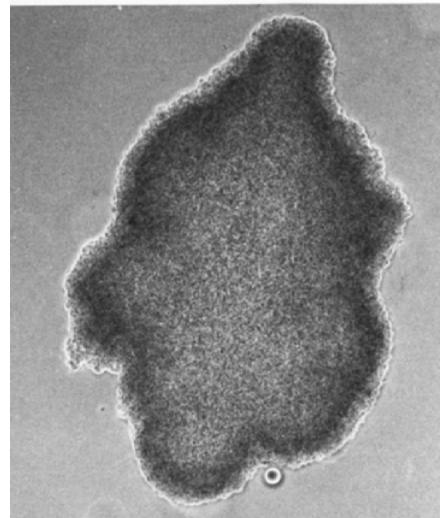
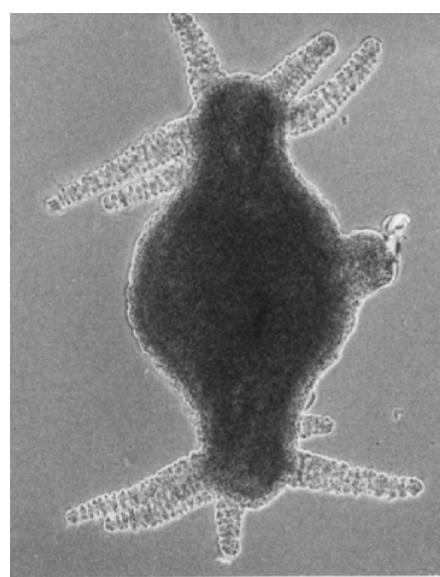
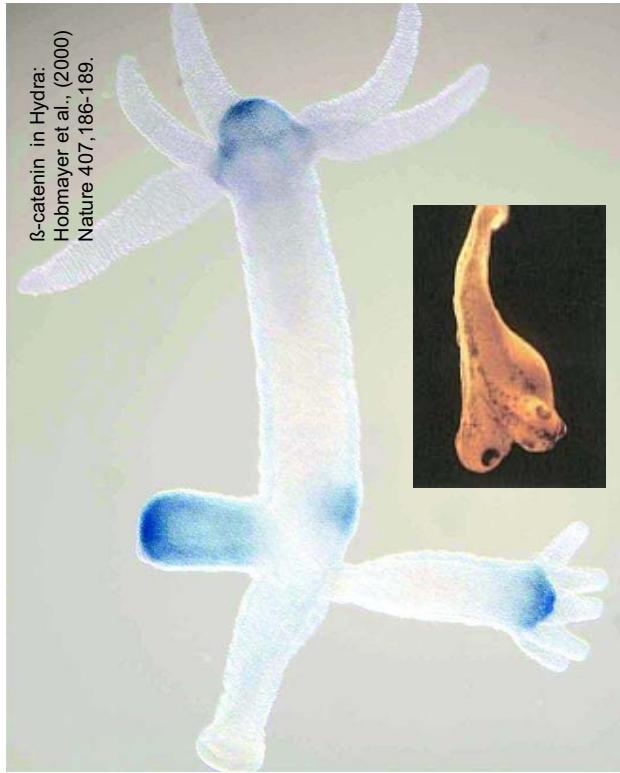
# Elementary networks for pattern formation in early development

Hans Meinhardt  
Max-Planck-Institut für Entwicklungsbiologie  
Tübingen

**Minimal models:** what is required at least

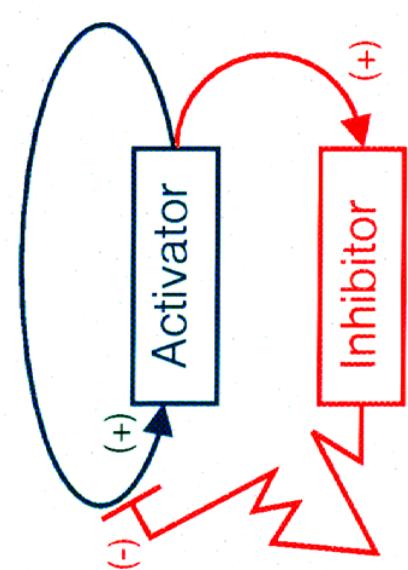
Complementary approach:

# An ancestral axis: pattern formation in hydra



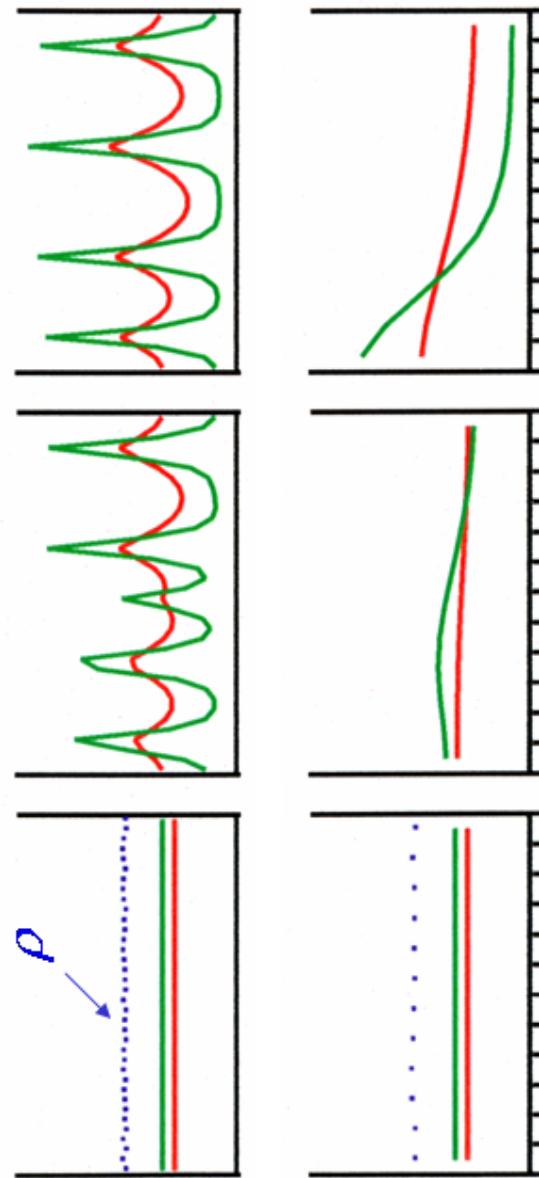
Gierer A, Berking S, Bode H, David C.N., Flick K., Hansmann G., Schaller H. and Trenkner E. (1972). Regeneration of hydra from reaggregated cells. Nature New Biology 239, 93-101. Figure: Holstein-group

# Pattern formation requires local self-enhancement and long range inhibition (with Alfred Gierer, 1972)



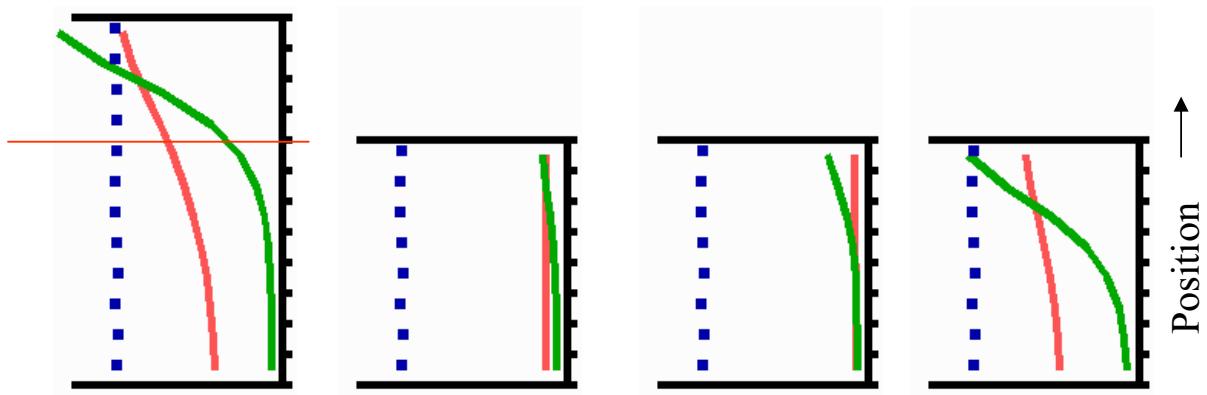
$$\frac{\partial \textcolor{green}{a}}{\partial t} = \rho \frac{\textcolor{green}{a}^2}{\textcolor{red}{h}} - \mu_a \textcolor{green}{a} + D_a \frac{\partial^2 \textcolor{green}{a}}{\partial x^2} + \rho_a$$

$$\frac{\partial \textcolor{red}{h}}{\partial t} = \rho \textcolor{green}{a}^2 - \mu_h \textcolor{red}{h} + D_h \frac{\partial^2 \textcolor{red}{h}}{\partial x^2} + \rho_h$$

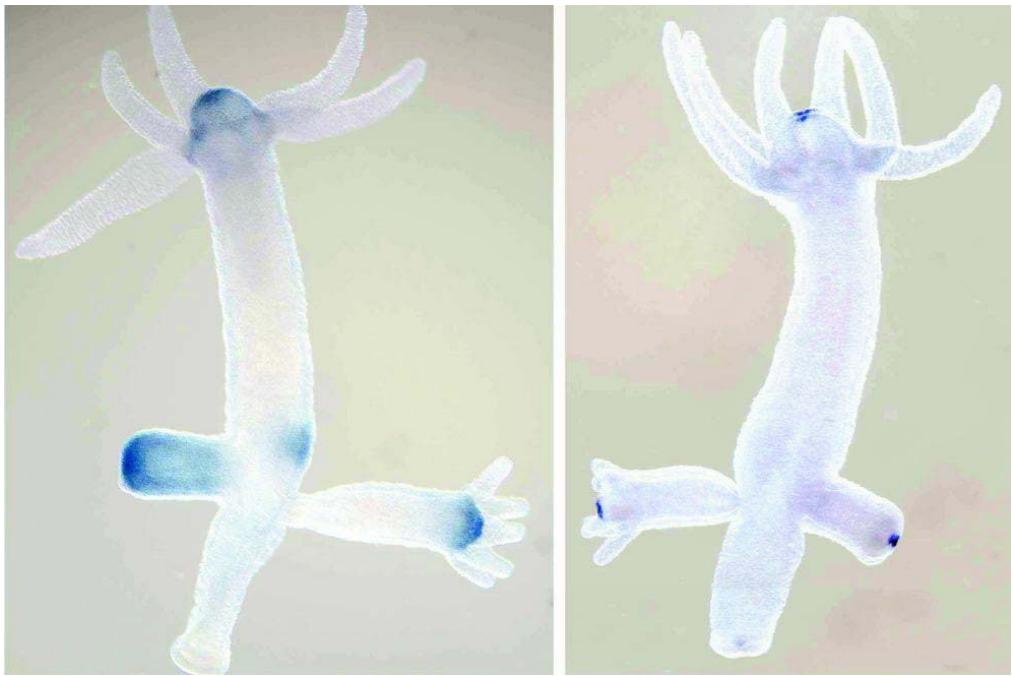
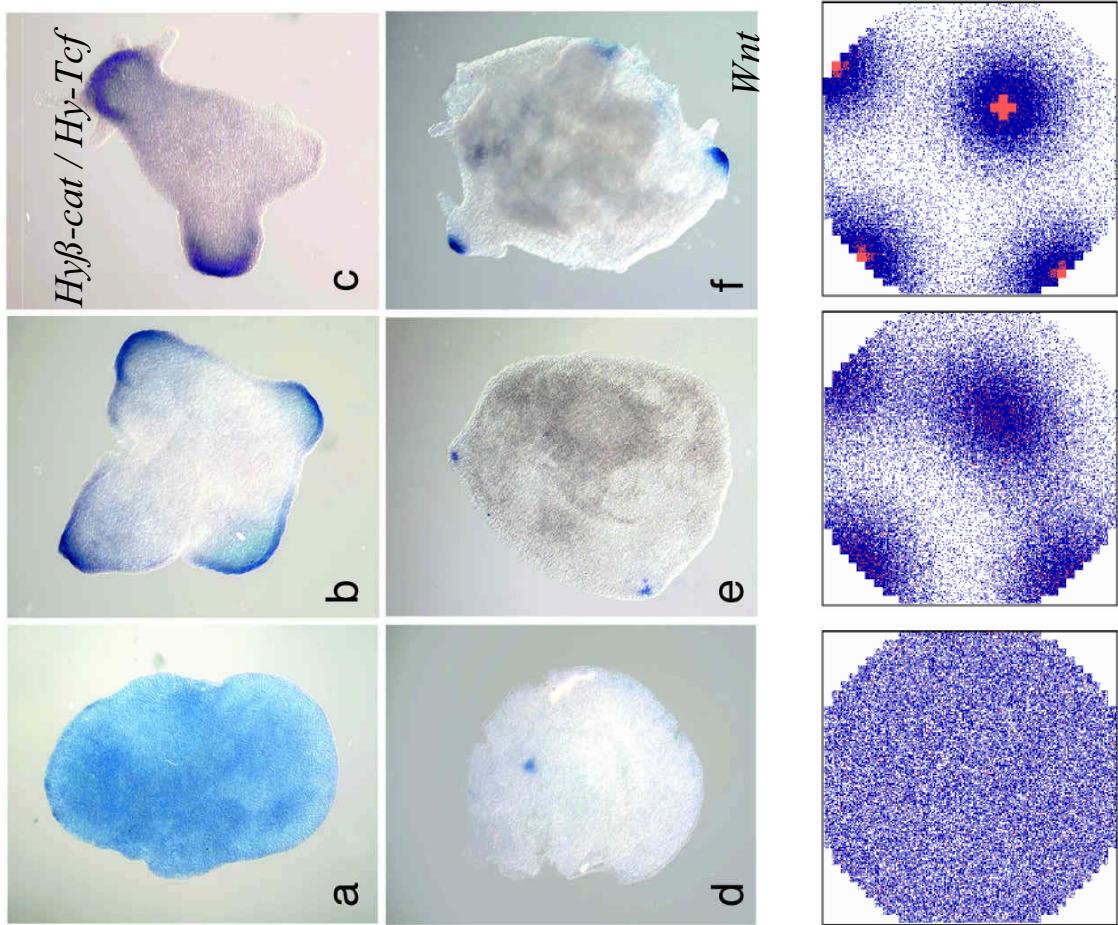


Kybernetik 12, 30-39 (1972);  
(on our web-site)

Regeneration of a  
graded distribution:  
inhibitor decays, a new  
activation is triggered

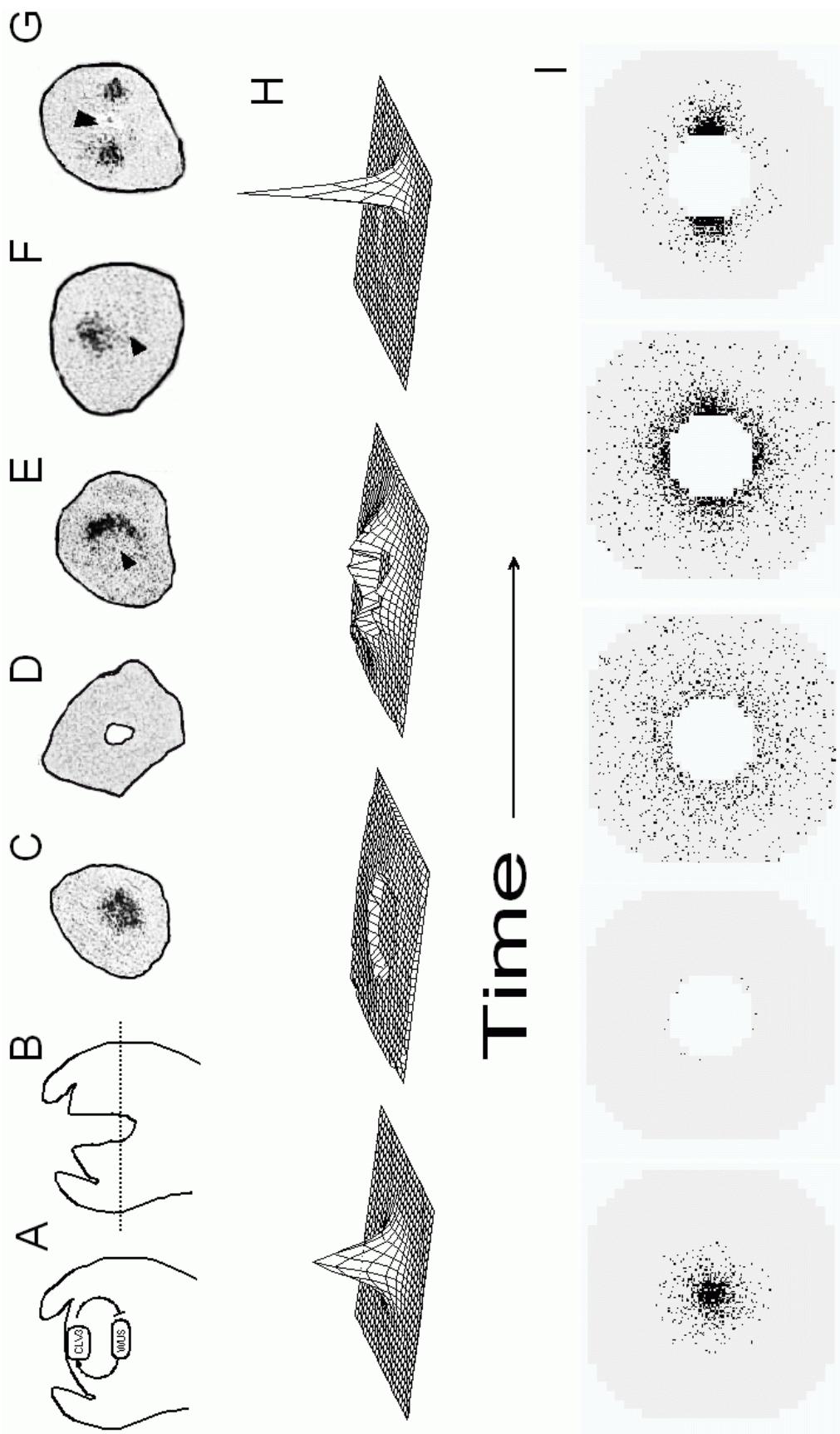


# *HyTcf* and *HyWnt* in aggregates



Hobmayer et al.: 2000  
Nature 407, 186-189 (2000)

After laser ablation of the plant organizing region (Wuschei)  
one or two organizing regions regenerate



A-G after Reinhardt et al., (2003)  
Development 132, 15-26

- ...If the range of the inhibitor is smaller than the field size:  
periodic patterns

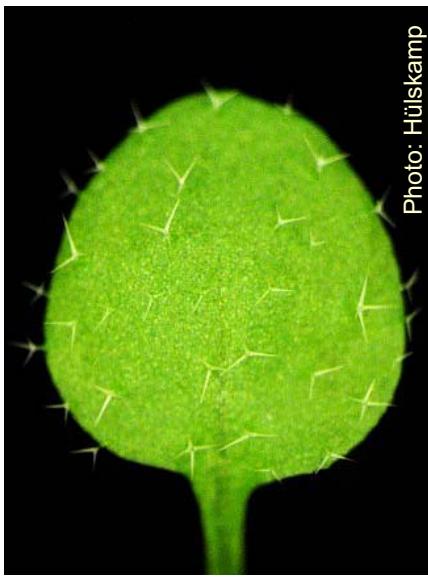


Photo: Hülskamp

Prediction: Activator and Inhibitor are produced at the same position

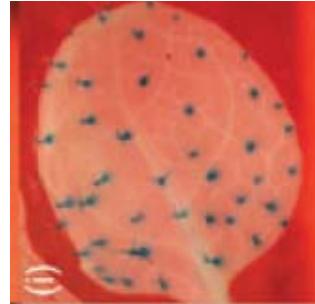
An aktivator (*Gl2*)

an Inhibitor (*try*)

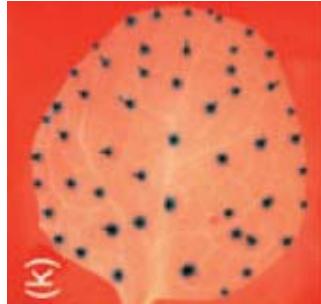
$$\frac{\partial \textcolor{green}{a}}{\partial t} = \rho \frac{\textcolor{green}{a}^2}{\textcolor{red}{h}} - \mu_a \textcolor{green}{a} + D_a \frac{\partial^2 \textcolor{green}{a}}{\partial x^2} + \rho_a$$

$$\frac{\partial \textcolor{red}{h}}{\partial t} = \rho \textcolor{blue}{a}^2 - \mu_h \textcolor{red}{h} + D_h \frac{\partial^2 \textcolor{red}{h}}{\partial x^2} + \rho_h$$

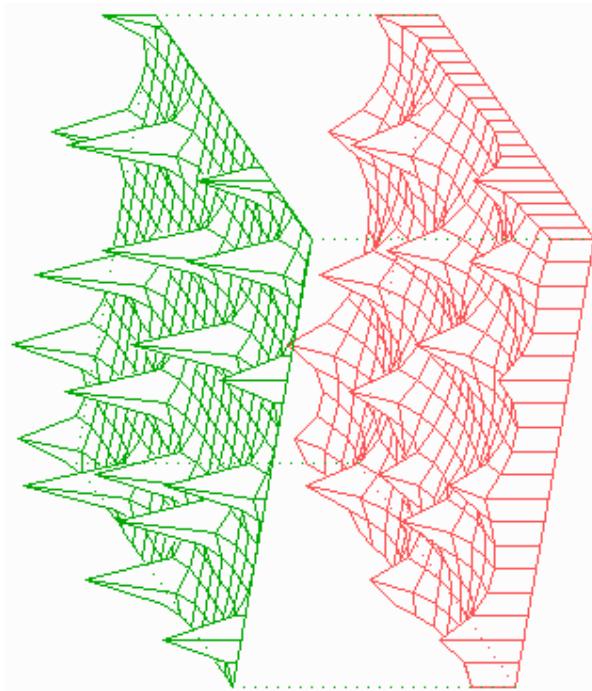
From: Esch et al,  
Plant J. 40, 860, 2004



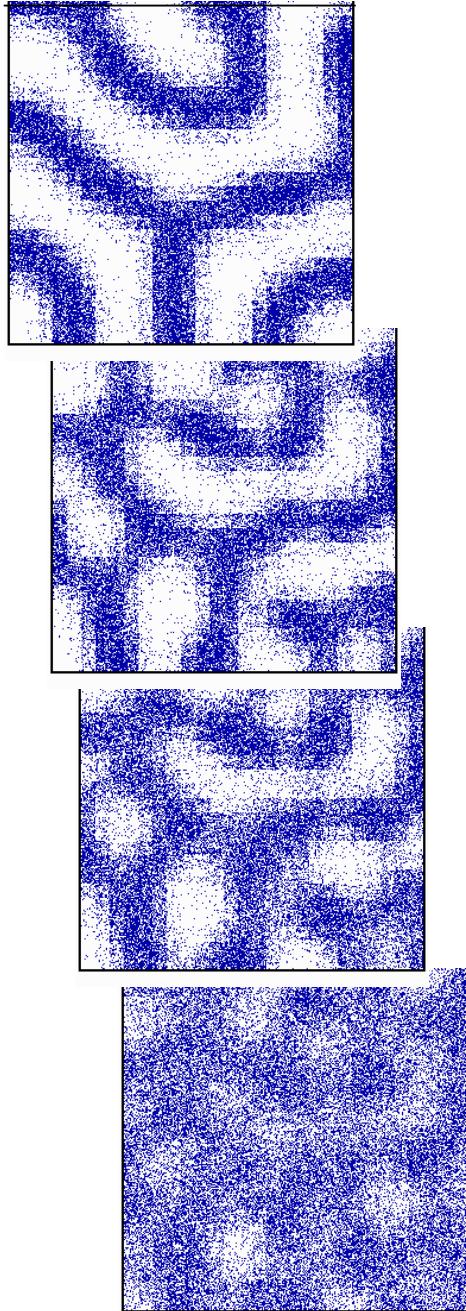
(l)



(k)

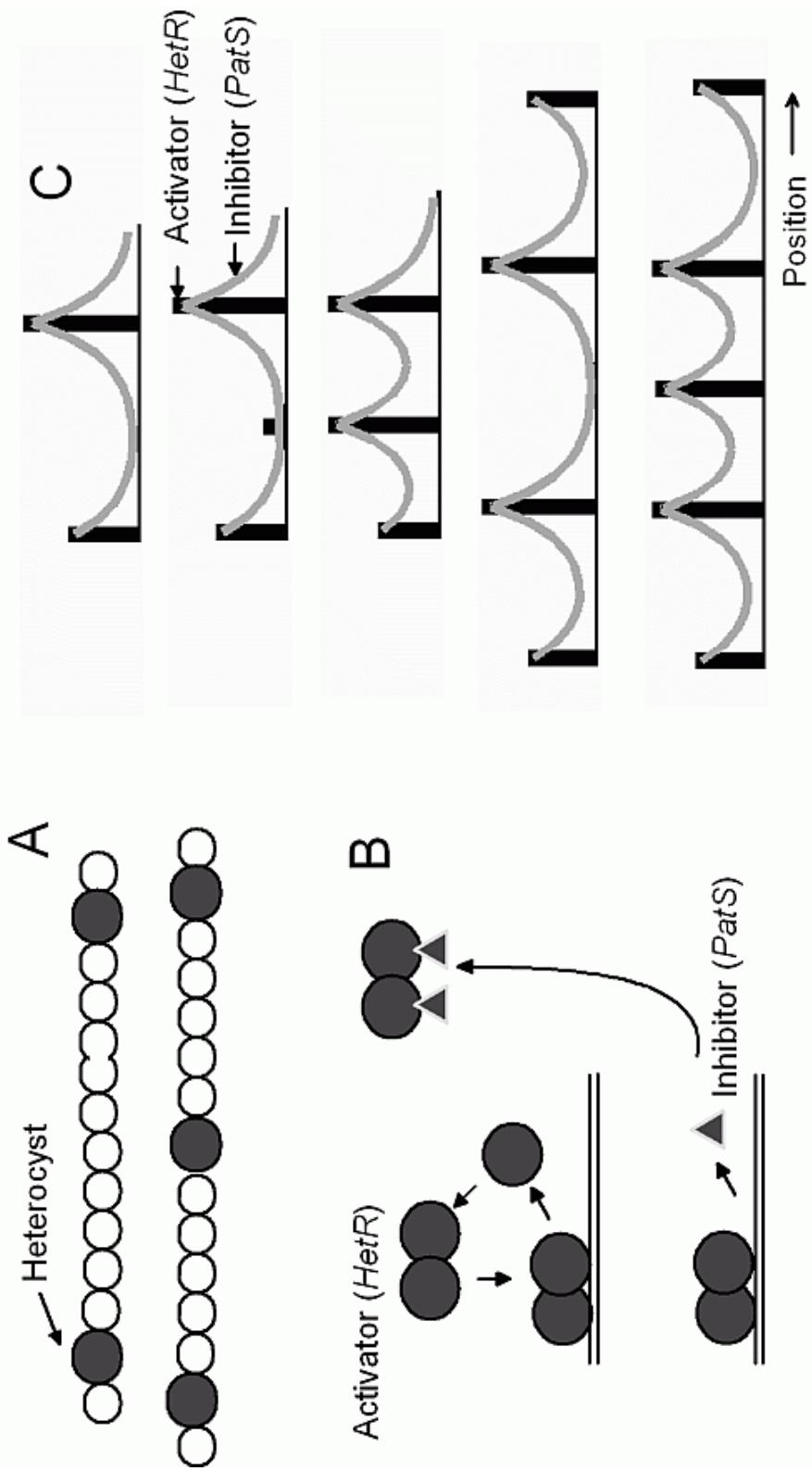


## Saturation of the autocatalysis: stripe formation



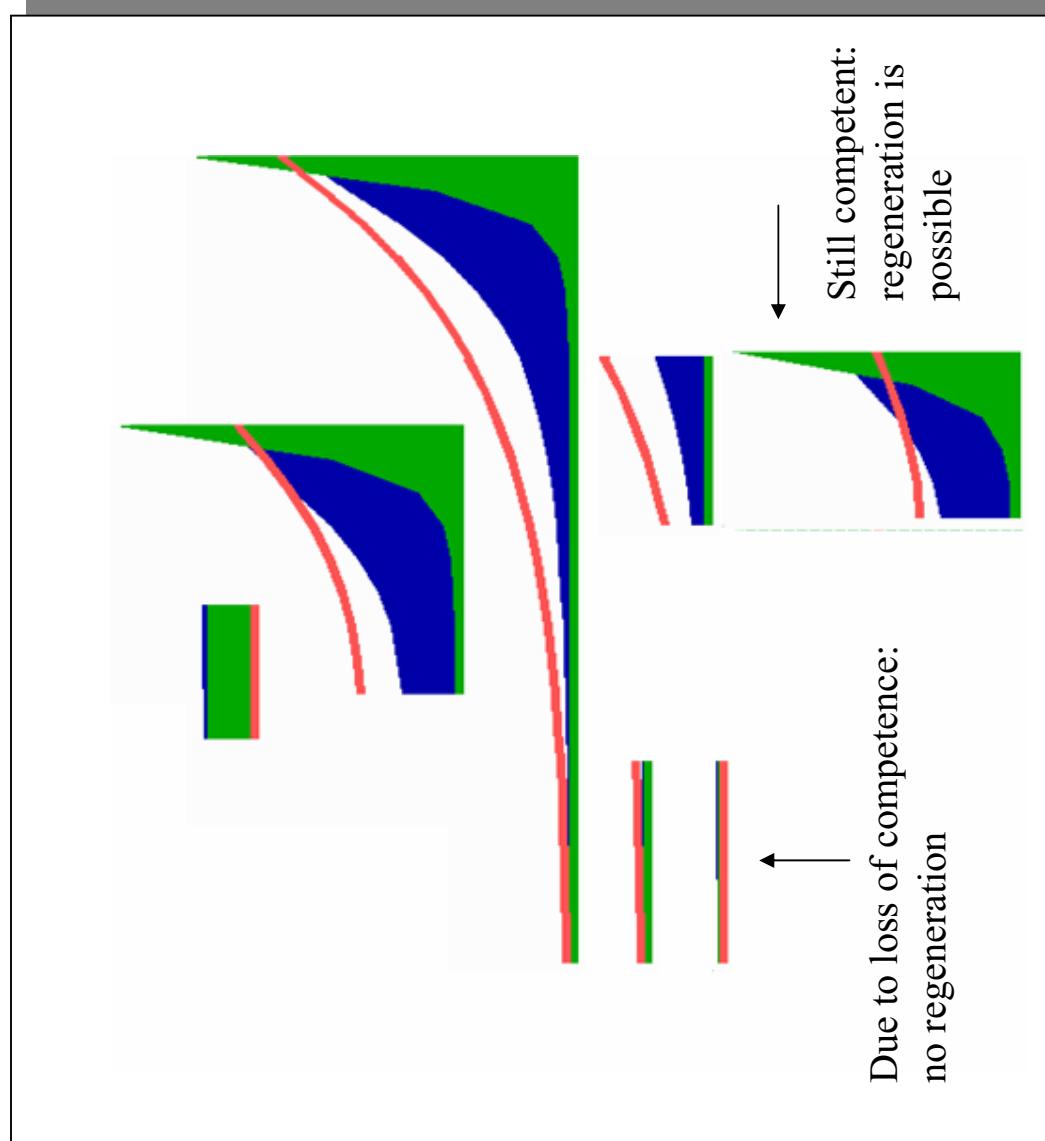
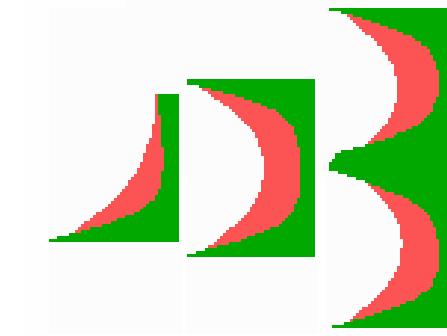
$$\frac{\partial a}{\partial t} = \frac{s a^2}{b(1 + s_a a^2)} - \dots$$

# Heterocyst formation in Anabaena: new Nitrogen-fixating cells are initiated during growth by an activator-inhibitor system



Activator: HetR, a DNA-binding molecules that forms dimers (Nonlinearity)  
Inhibitor: PatS, a 13-17 AA polypeptide, binds to HetR, abolishes its DNA binding

The wavelength problem: how to avoid multiple peaks during outgrowth?



$$\frac{\partial a}{\partial t} = \frac{\rho a^2}{h} - \dots$$

$$\frac{\partial h}{\partial t} = \rho a^2 - \dots$$

$$\frac{\partial \rho}{\partial t} = c a - \gamma \rho \dots$$

Solution: feedback of the organizer on the competence

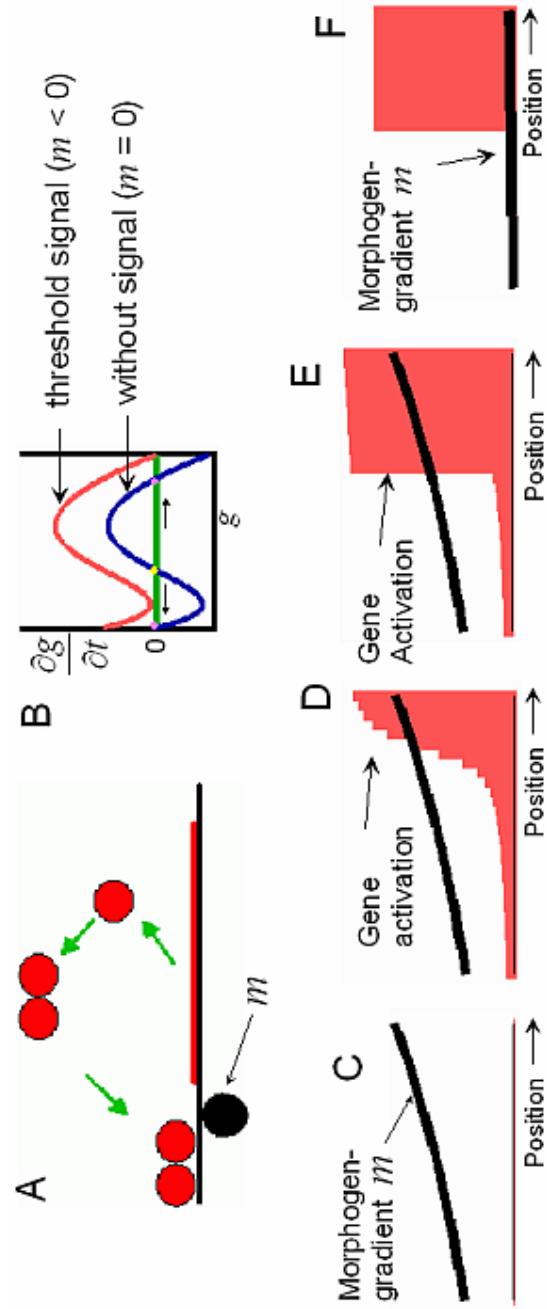
Thus:

Pattern formation requires local self-enhancement and  
long-range inhibition

Next:

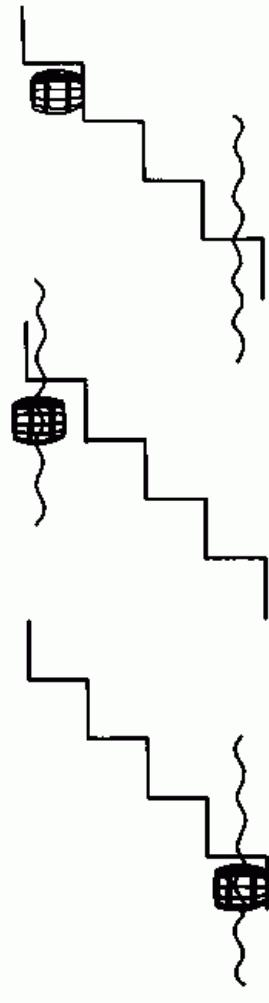
How to achieve stable gene activation?

# Gene activation requires autoregulatory genes



Differentiation 6, 117-123 (1976)

## Space-dependent activation of several genes by a morphogen gradient: step-wise promotion

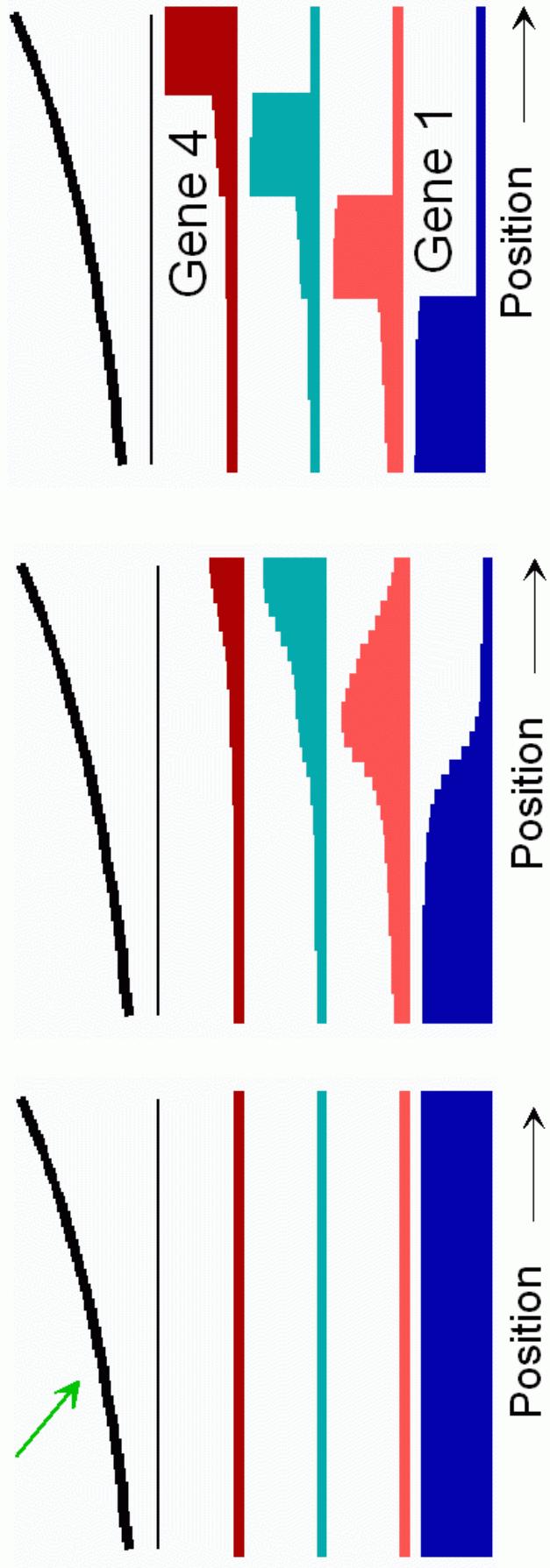


$$\frac{\partial g_i}{\partial t} = \frac{c_i g_i^2 + b_i g_{i-1} m}{\sum_{i=1}^n c_i g_i^2} - r_i g_i$$

with  $c_{i+1} > c_i$

Morphogen gradient ( $m$ )

J. theor. Biol. 74,307-321



Thus:

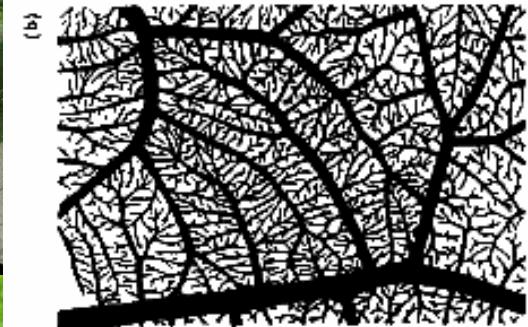
Stable gene activation requires an autoregulatory feedback of a gene product on its own gene

Competition between alternative genes leads to unequivocal decisions

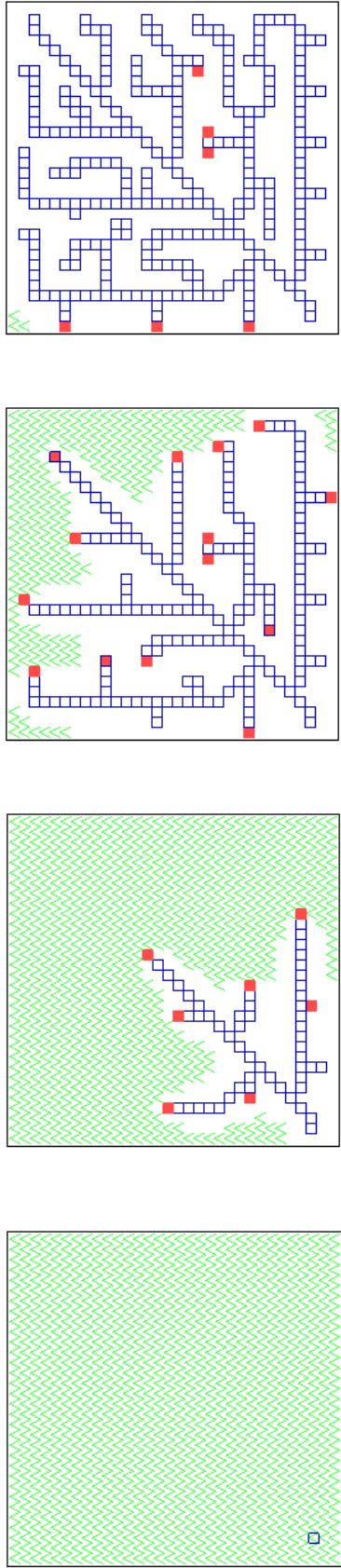
'Promotion'

Next:

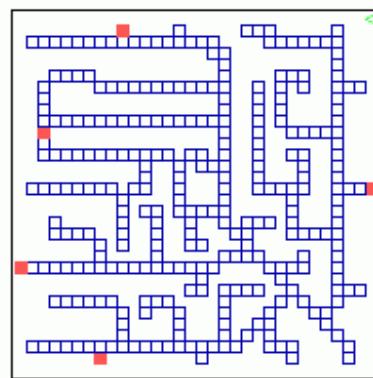
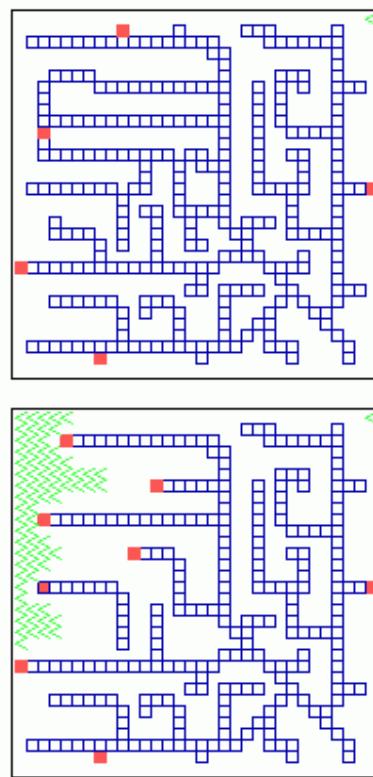
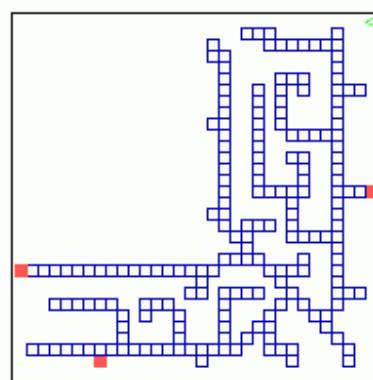
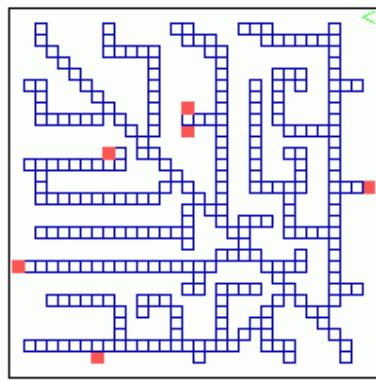
How to achieve branched structures



**Formation of a netlike structure:  
a trace behind a shifting signal**

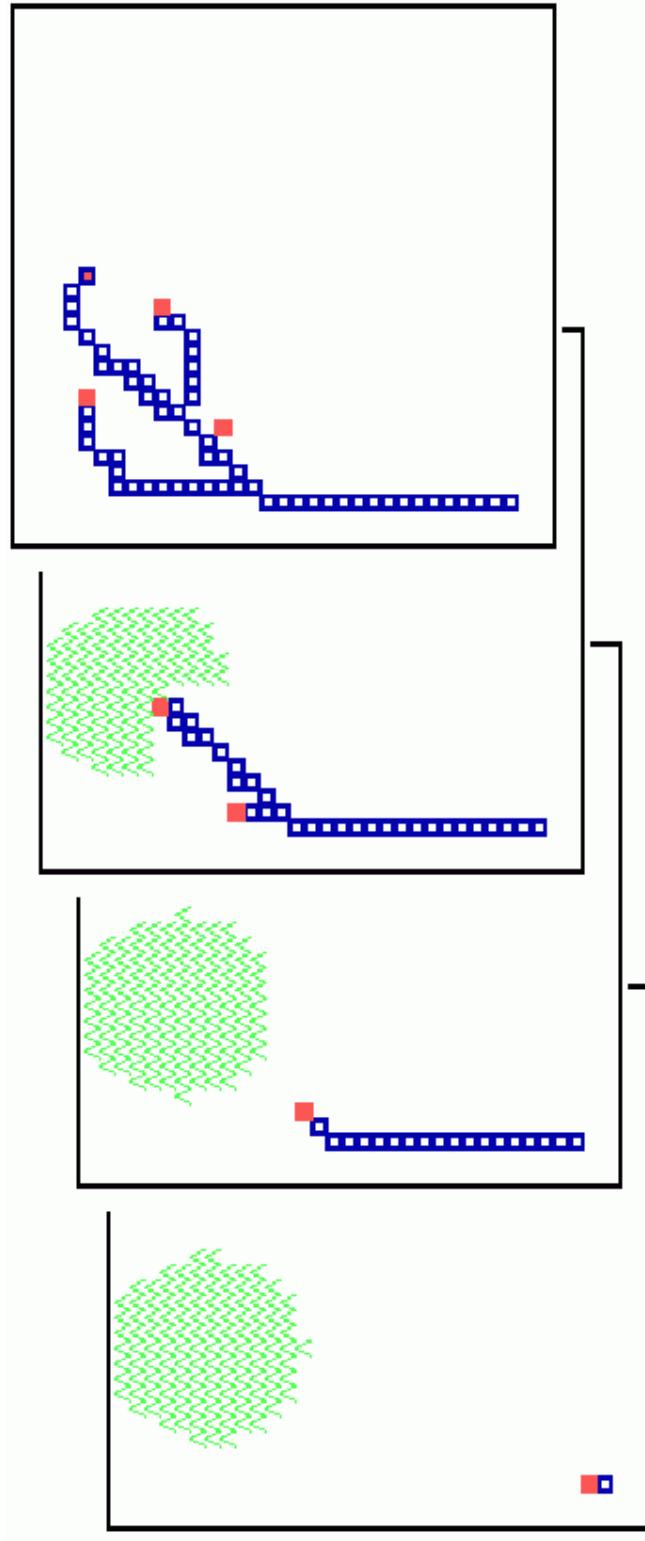


# Regeneration of a net-like structure



Differentiation 6, 117-123 (1976)

# Elongation towards a target area



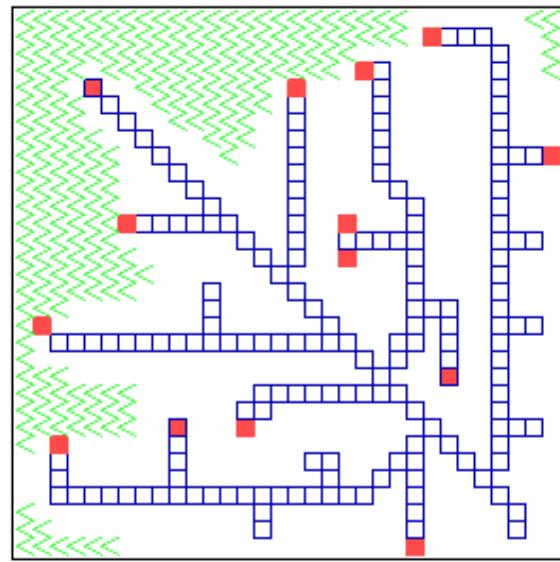
J. Cell Sci. 112, 2867-2874 (1999)  
Differentiation 6, 117-123 (1976)

Thus, for net-like structures we need:

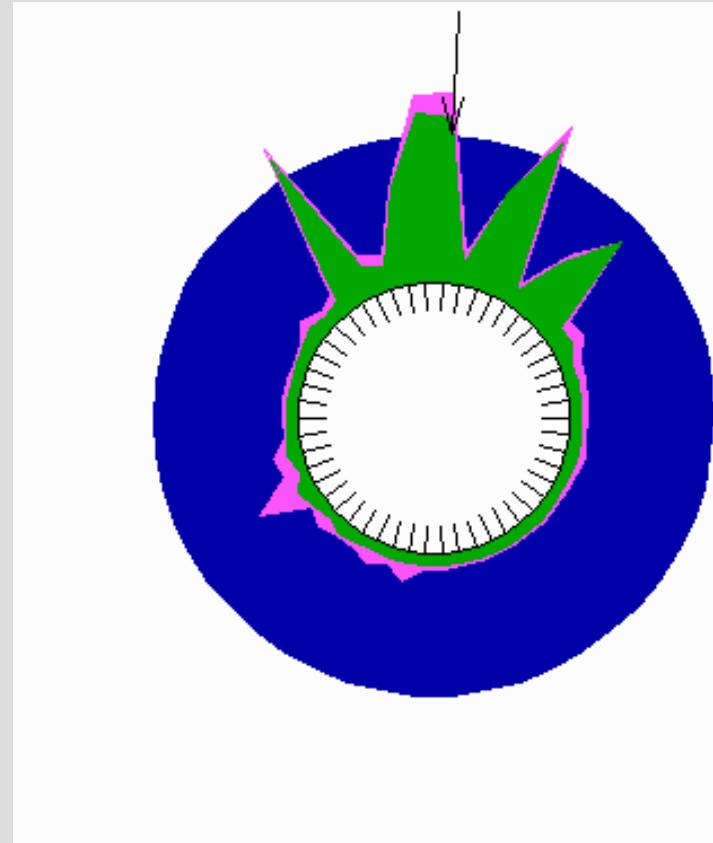
1. A signal that determines at which position the filament should be elongated or where to initiate a new branch (e.g., an activator-inhibitor system)
2. The filaments remove something (auxin, NGF, oxygen deficiency)

3. A irreversible determination that makes the filament different from the remaining cells

$$\begin{aligned}\frac{\partial a}{\partial t} &= \frac{sca^2}{b} - r_a a + D_a \Delta_a + b_a d \\ \frac{\partial b}{\partial t} &= sca^2 - r_b b + D_b \Delta b + b_b d \\ \frac{\partial c}{\partial t} &= b_c - r_c c - c_c c d + D_c \Delta c \\ \frac{\partial d}{\partial t} &= \frac{r_d d^2}{1 + s_d d} - r_d d + b_d a\end{aligned}$$



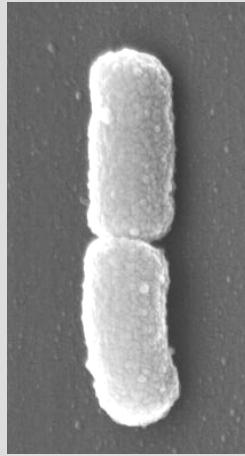
Permanent sensitivity is achieved by generation of local signals and their subsequent local destabilization



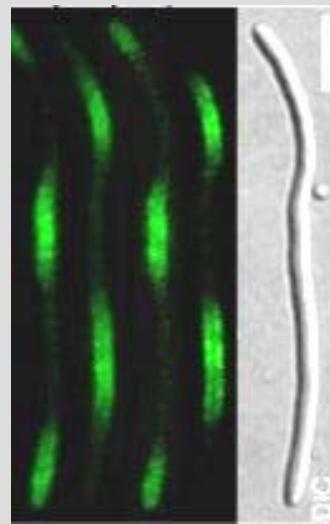
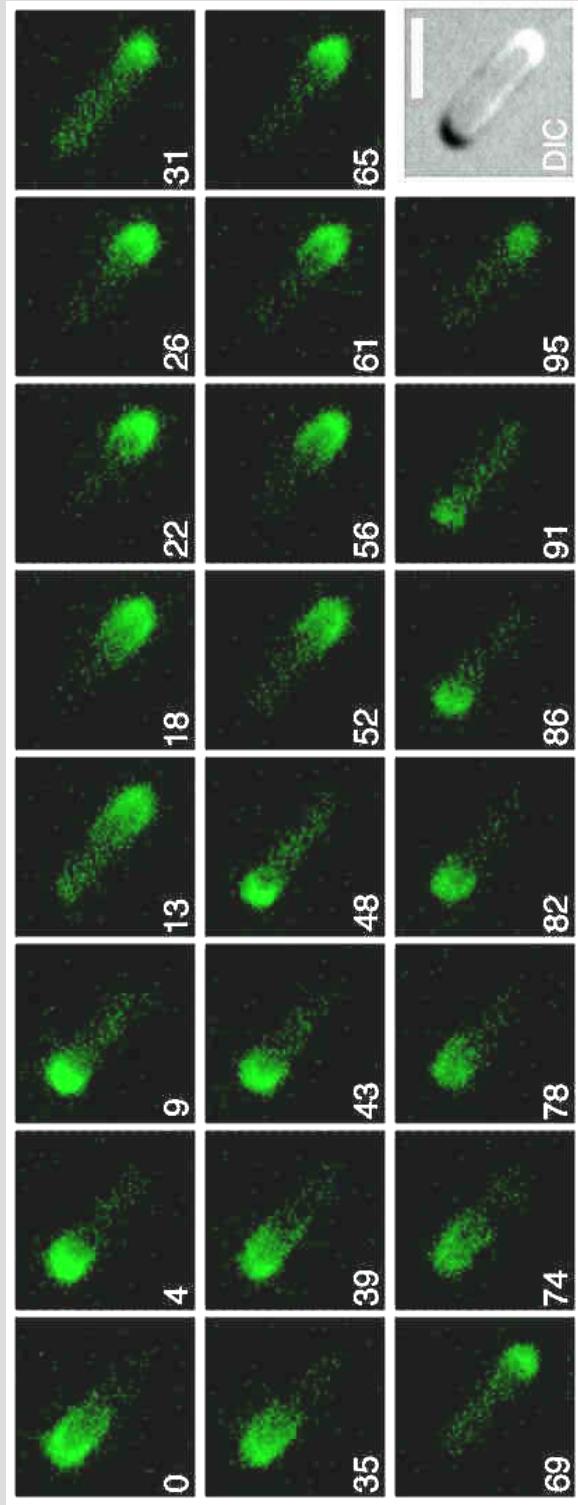
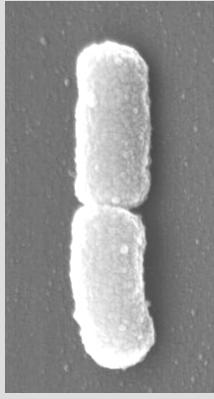
(separation of inhibition in time and inhibition in space)

J. Cell Sci. (1999) 112, 2867-2874

The use of local destabilization and  
waves in intracellular pattern formation:  
Center finding in E.coli

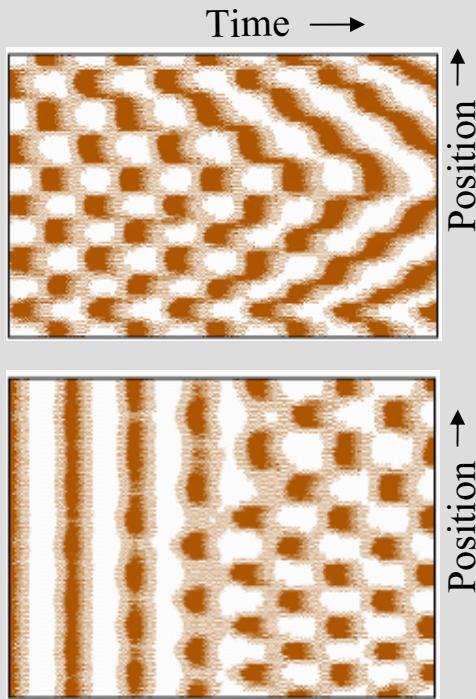


# Oscillation of *MinD* out of phase



Raskin and de Boer (1999):  
J. Bacteriology 181,6419-6424

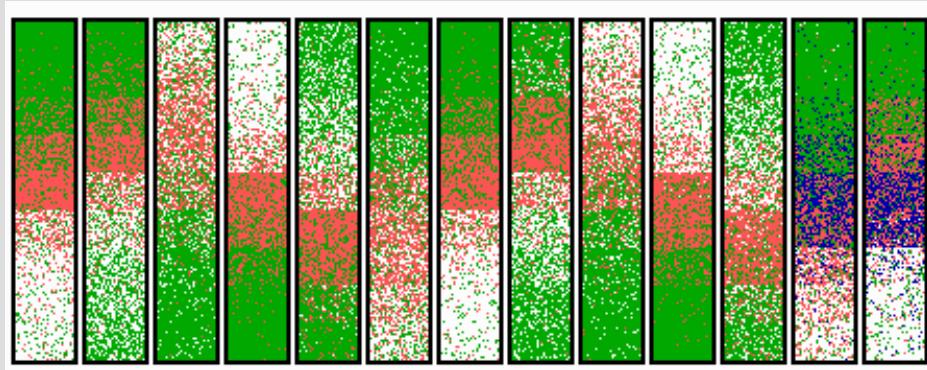
The key: oscillations in counter phase are generated by destabilizations of recently formed peaks



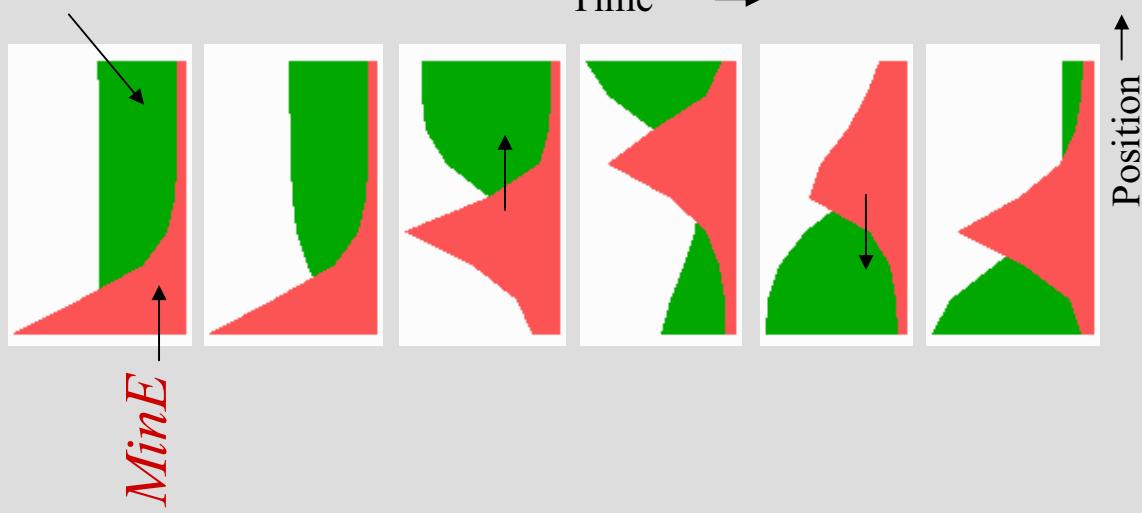
Meinhardt and Klingler (1987)  
J. theor. Biol. 126, 63-69  
H.M., Algorithmic Beauty of Sea Shells  
(Springer-Verlag, 1995 / 2003)

# MinD pole-to-pole oscillation by MinE removal

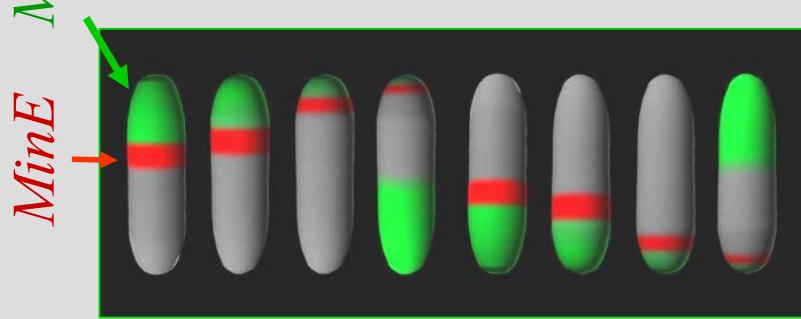
*MinD*



Time →



*MinE* *MinD*



Meinhardt and de Boer (2001). PNAS 98,14202-14207

↑ *FtsZ*

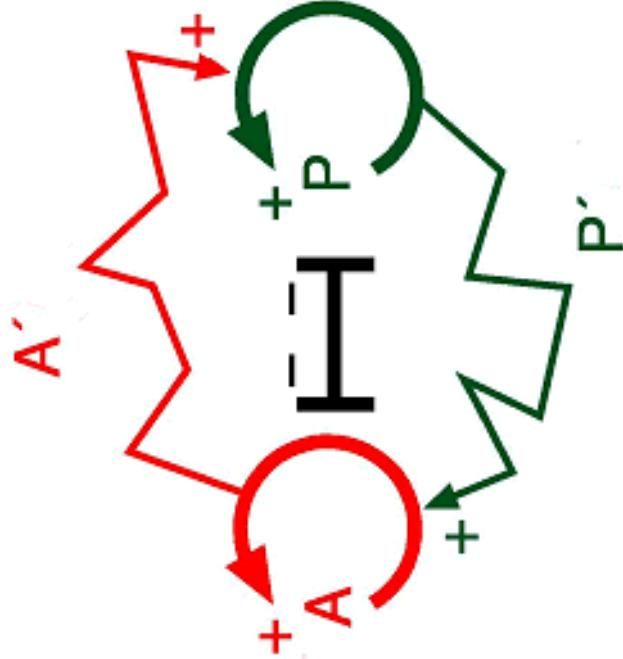
Next: Segmentation:

Mutual activation of structures that locally exclude each other

# Mutual long range activation of locally exclusive cell states: Segmentation as example



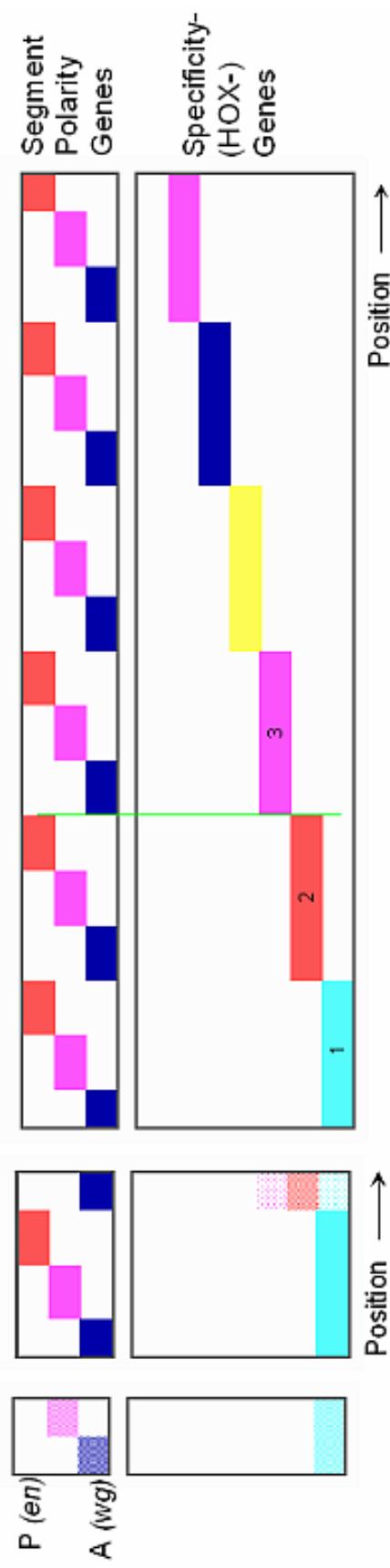
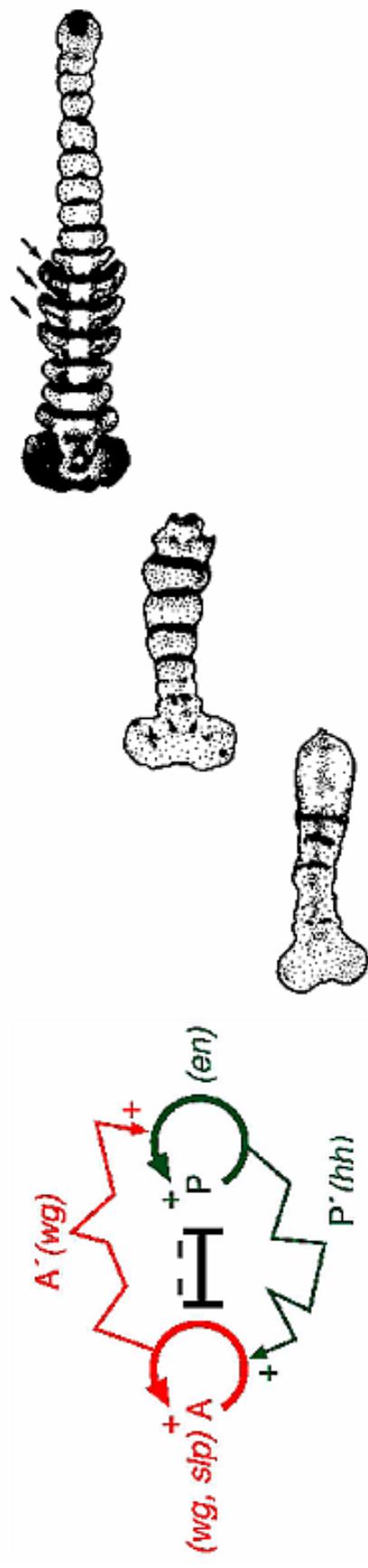
after N. Patel



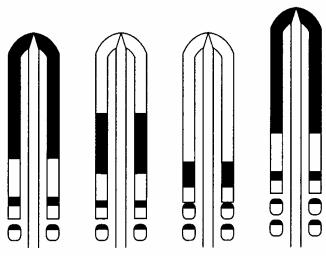
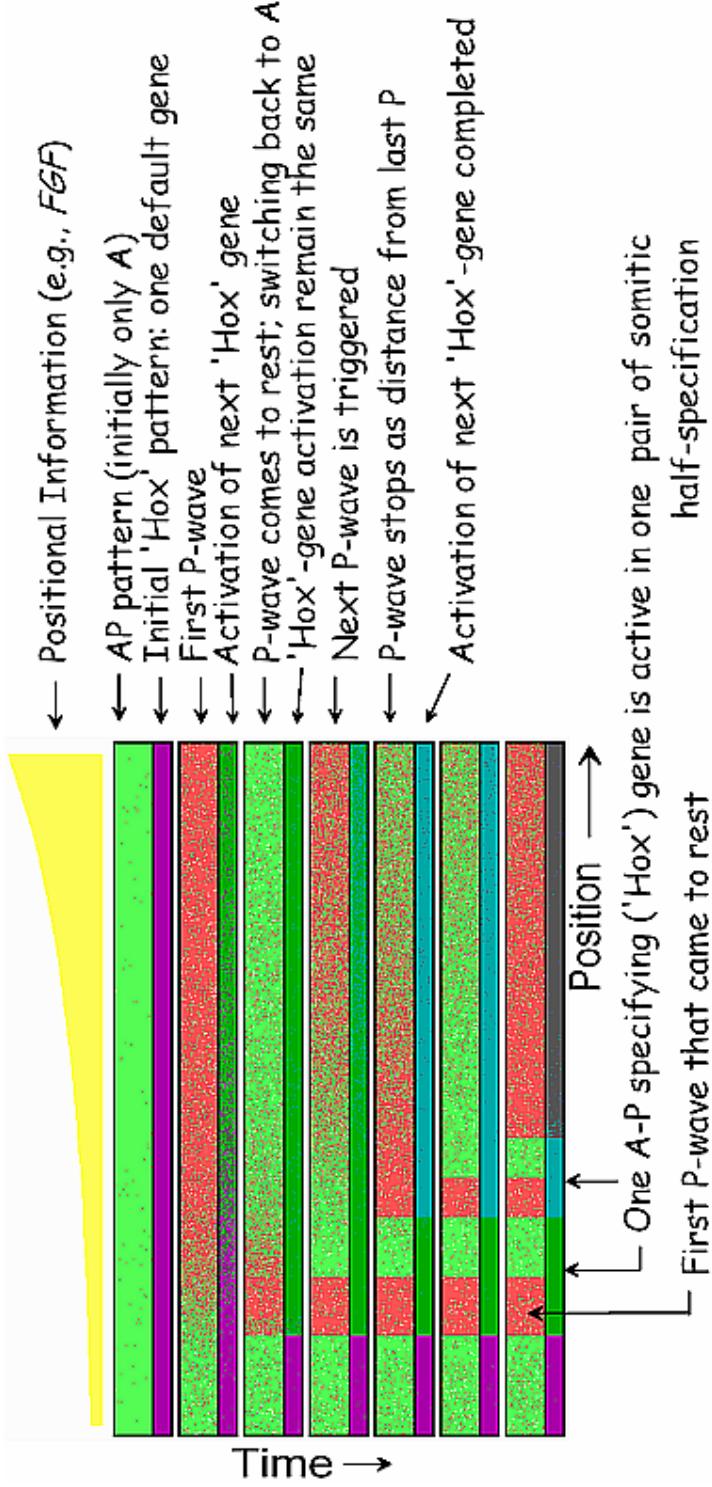
$$\begin{aligned}
 \frac{\partial a}{\partial t} &= \frac{cs_p a^2}{r} - \alpha a + D_g \frac{\partial^2 a}{\partial x^2} + \rho_0 \\
 \frac{\partial p}{\partial t} &= \frac{cs_a p^2}{r} - \alpha p + D_g \frac{\partial^2 p}{\partial x^2} + \rho_0 \\
 \frac{\partial s_a}{\partial t} &= \gamma(a - s_a) + D_s \frac{\partial^2 s_a}{\partial x^2} + \rho_1 \\
 \frac{\partial s_p}{\partial t} &= \gamma(p - s_p) + D_s \frac{\partial^2 s_p}{\partial x^2} + \rho_1 \\
 \frac{\partial r}{\partial t} &= cs_p a^2 + cs_a p^2 - \beta r \left[ + D_r \frac{\partial^2 r}{\partial x^2} \right]
 \end{aligned}$$

Meinhardt and Gierer (1980) J. theor. Biol., **85**, 429-450

Segmentation: Mutual long range activation of locally exclusive cell states generates a periodic structure; oscillation at the posterior pole allows a counting in the gene level



Somite formation: a sequential conversion of a periodic pattern in time into a periodic pattern in space; correctly predicted: waves move from posterior towards anterior and form anterior and posterior half-somites



Models of Biological Pattern Formation,  
Academic Press, 1982 (available on our website)

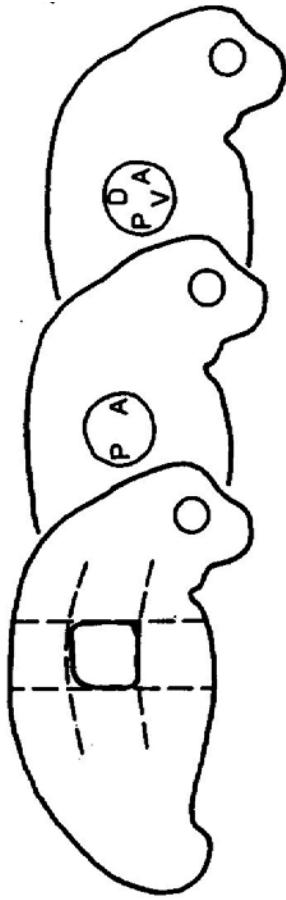
Oscillation of *c-hairy1* in the chick  
Palmeirim et al. (1997). Cell 91, 639-648

**Next:**

**Initiation of legs, wings...**

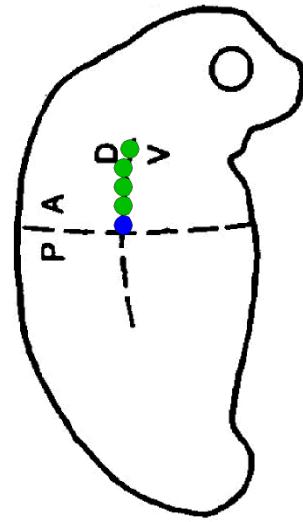
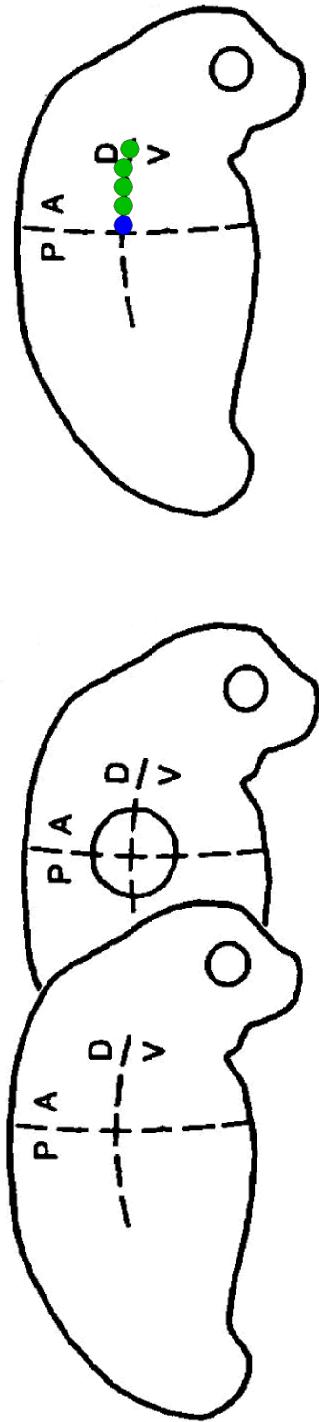
**Intersection of boundaries become the new  
organizing regions**

## Initiation of legs and wings

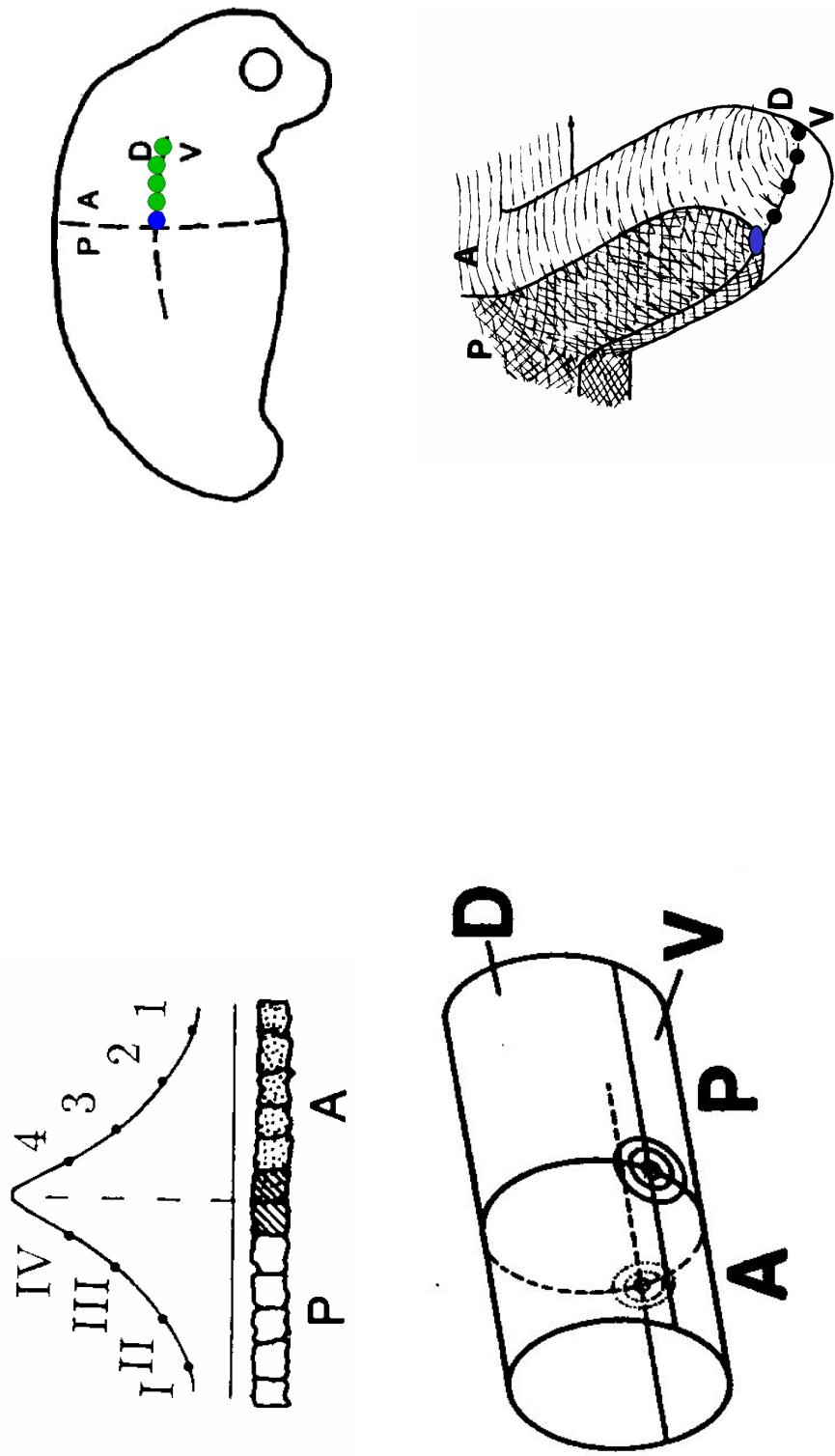


Conventional view: (G. Harrison, 1918)  
Homogeneous limb field that becomes  
subsequently subdivided

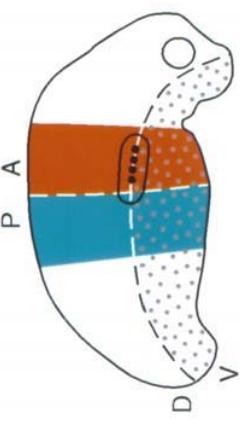
Proposal (1980,1983):  
Differentiation borders first



Vertebrate limb: only the anterior part is used

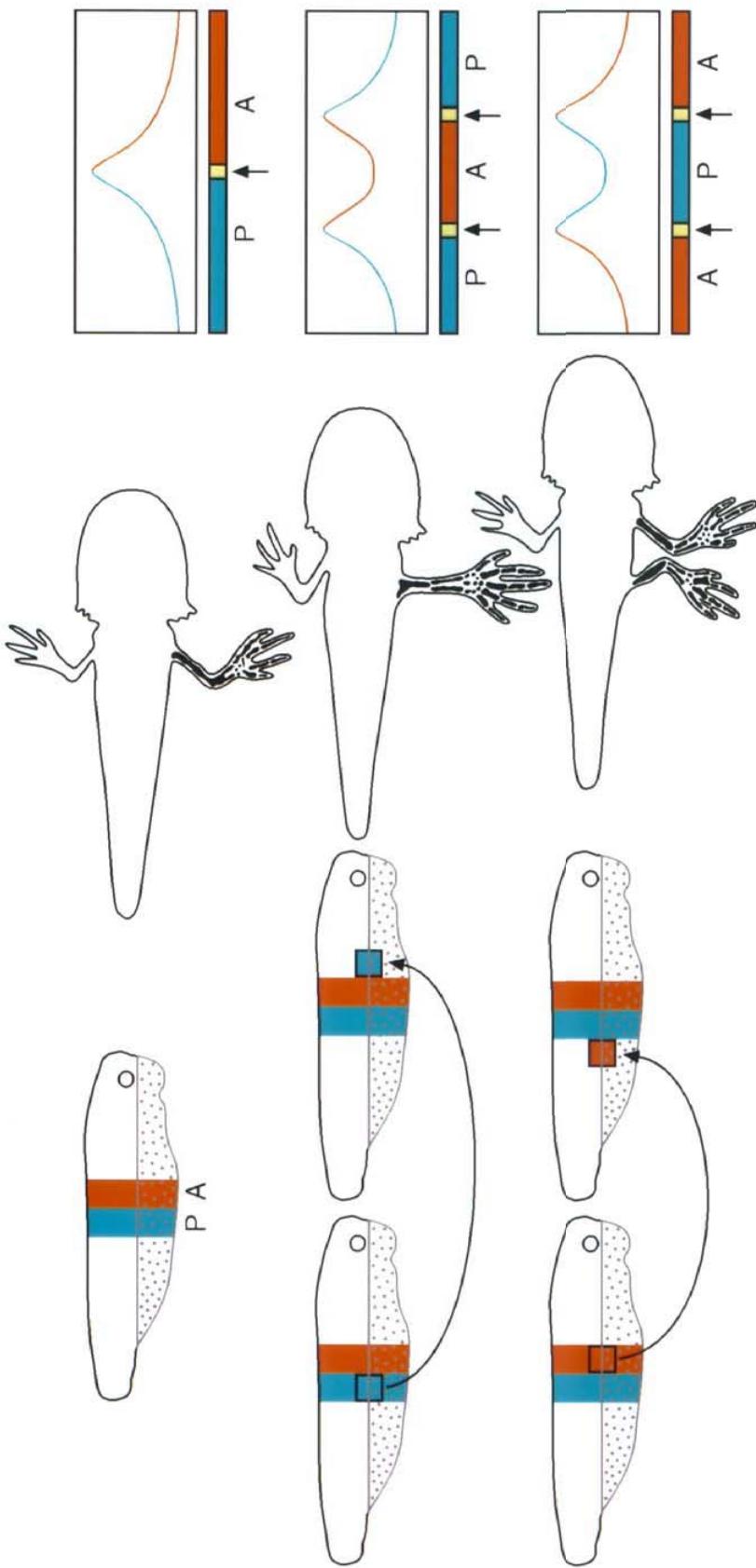


H.M. (1983) A boundary model for pattern formation in  
vertebrate limbs. *J. Embryol. exp. Morphol.*, 76, 115-137

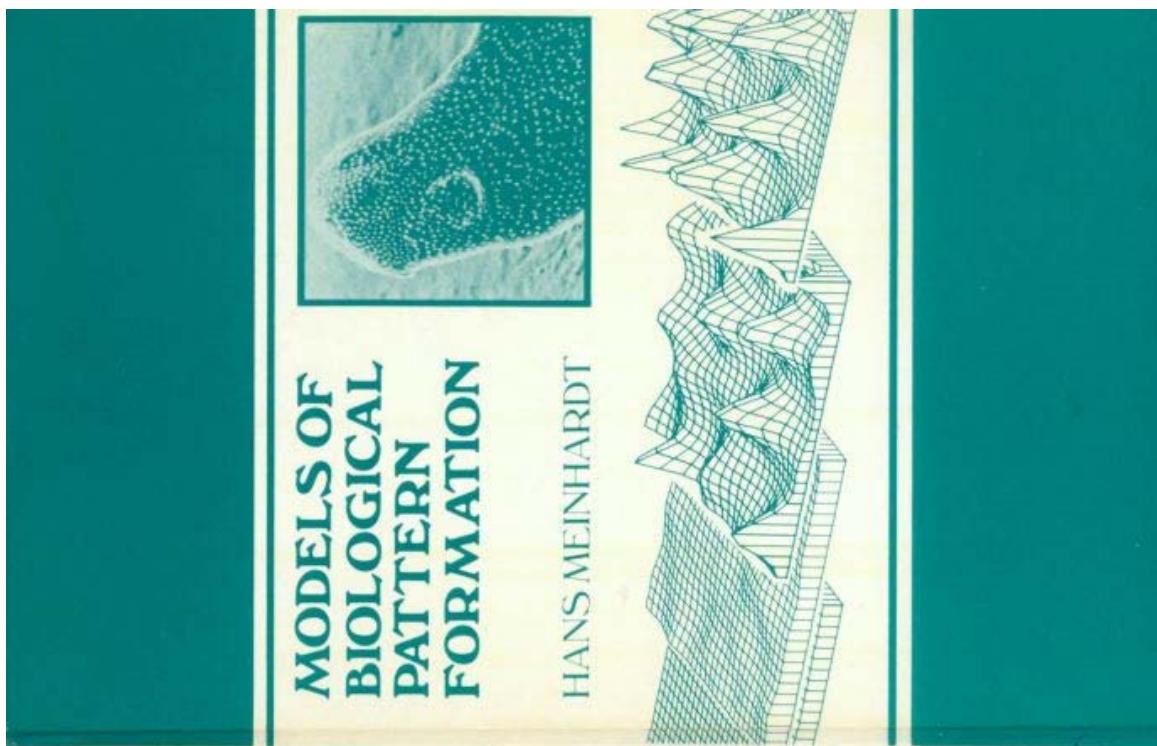


## Slack's experiments

(Nature 261, 44-46; 1976)



Model: A boundary model for pattern formation in vertebrate limbs  
J. Embryol. exp. Morphol. 76, 115-137 (1983)



Academic Press (1982);  
available at  
<http://www.eb.tuebingen.mpg.de/meinhardt>

